containing 0.75 lb of propellant. Comparison of the burning rate of an ammonium perchlorate propellant based on PPG2025/Z-6018 copolymer glycol polyurethane binder containing 2.07% silicon with that of a conventional polyurethane propellant based on PPG2025 instead of the copolymer glycol, but otherwise identical, showed burning rate to be increased from 0.20 in./sec to 0.575 in./sec at 1000 lb/in.^{2} by the inclusion of silicon. The copolymer glycols derived from higher molecular weight PPG had higher viscosities than the PPG2025/Z-6018 copolymer glycol and were less useful for preparing propellants.

Although any argument can only be speculative in explaining the effectiveness of Z-6018 in increasing (by 188%) the burning rate of polyurethane propellant compared with the much smaller increases observed by Schwarz and Lowrey for silane polysulfide propellants (48% increase for binder containing 2.14% silicon; 52% increase for 8.1% silicon), the bonding of oxygen to silicon before combustion provides the most attractive explanation, paralleling the effect of elemental metals vs metal oxides on burning rate. In the case of the silane moiety, oxidation of silicon may occur too late during combustion to effect maximum enhancement of burning rate.

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Breakup of a Liquid Jet in a High-Pressure Environment

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Nomenclature

a = liquid to gas density ratio
b_i = mixing layer thickness
C_d = concentration of discharge fluid $\dot{m}_d/(\dot{m}_d + \dot{m}_e)$
f = velocity distribution function
g = concentration distribution function
L_b = jet breakup length
$\dot{m}_d$ = mass flow rate of discharge fluid
$\dot{m}_e$ = mass flow rate of entrained fluid
r = radial coordinate
r_0 = orifice radius
r_1 = location of inner boundary of mixing layer
r_2 = location of outer boundary of mixing layer
U = mean axial velocity
U_0 = axial discharge velocity
$\bar{U}$ = $U/\bar{U}_0$
ΔU_c = critical velocity difference for secondary atomization
$\Delta \bar{U}_c$ = $\Delta U_c/U_0$
x_i = length of initial region
$\bar{x}_i$ = x_i/r_0
η = $(r_2 - r)/b_i$
η_0 = $(r_2 - r_0)/b_i$

Liquid issuing from an orifice into gaseous surroundings will always atomize, i.e., disintegrate into liquid droplets. An important parameter when describing the atomization process is the jet breakup or atomization length L_b. This is defined as the distance downstream from the orifice exit to the point where atomization is complete. Except for a few idealized cases,[1,2] it has not been possible to determine this distance analytically for any given liquid. This is simply because the atomization process is so very complex. However, the results of a recent study[3] indicate that under certain realistic circumstances it should be possible to ascertain the value of this parameter.

The study in question was concerned with the behaviour of propellant sprays in high-pressure combustors. Here it was observed that, in a large number of cases, the dynamic and geometric characteristics of the spray closely resembled those of the so-called turbulent submerged jet.

Shown in Fig. 1 is the customary representation of the inner and outer boundaries of the initial region of such a jet. The situation depicted here usually arises when a fluid exits from a circular orifice into a large quiescent medium of the same phase (i.e., liquid into liquid, gas into gas). In such cases there are three identifiable regions. The core, bounded by the surface $\eta = 1$, consists entirely of discharge fluid moving at velocity U_0. The region beyond the outer boundary $\eta = 0$, is totally environmental fluid and has zero axial velocity. The mixing layer bounded by $\eta = 0$ and $\eta = 1$ is comprised of discharge and environmental fluid with continuous distributions of velocity and mass concentration.

In a great many cases, the velocity and concentration profiles can be represented by equations of the form $U/U_0 = f(\eta)$ and $C_d = g(\eta)$. That is, the profiles are self preserving.

These are several functional forms available for $f(\eta)$ and $g(\eta)$. Two simple forms which closely approximate reality are given by[4,5]

$$f(\eta) = 2\eta^{3/2} + \eta^3 \tag{1}$$

and

$$g(\eta) = \eta \tag{2}$$

In the case of a liquid discharged into a high-pressure gas, the core region consists entirely of liquid and the ambient region is solely dense gas. The mixing region consists of atomized liquid droplets moving with the entrained gas; assuming of course that conditions are such that the atomization process is sufficient to produce a finely dispersed spray. When this is in fact the case, the length of the core region x_i represents quite well the breakup length. However before equating L_b to x_i, it is well to consider exactly what conditions must be met to insure the validity of this identity.

These are two mechanisms of atomization. Primary atomization is that form of breakup caused by deformation of the liquid stream at the liquid gas interface due to internal turbulence. Secondary atomization on the other hand, occurs

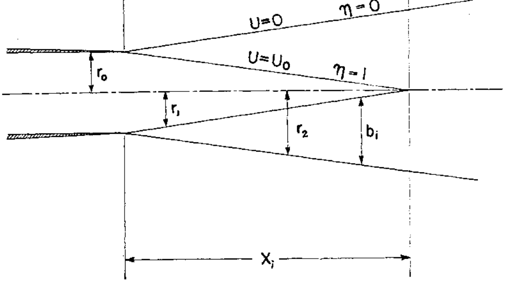

Fig. 1 Initial region of turbulent submerged jet.

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subsequent to the growth of the initial disturbances and can be regarded as the breaking up of the ligaments into many smaller droplets. This aerodynamic breakup phenomenon has been studied for example by Lane.[6] He found, as did others, that for a given surface tension and gas density, a liquid droplet would shatter when the relative velocity between the liquid and the gas exceeded some critical value ΔU_c.

Consider now the following: Assume that on the average the spray does behave in the manner depicted in Fig. 1. Assume further that the average velocity U at any point in the mixing layer is given by Eq. (1). Now the average velocity for any $1 < \eta < 0$ can be regarded as the total momentum flux per unit mass flow rate and can be expressed as

$$U = (\dot{m}_d U_d + \dot{m}_e U_e)/(\dot{m}_d + \dot{m}_e) \tag{3}$$

Using the definition of C_d, Eq. (3) can be written

$$U = C_d U_d + (1 - C_d) U_e \tag{4}$$

In that region of the mixing layer where secondary atomization has occurred, the droplets will be so small that they can be assumed to be moving with the entrained gas and hence $U_d \simeq U_e \simeq U$. However, close to (but less than) $\eta = 1$, there may be some fluid which has not reached the final stage of atomization. This assumption would certainly not be true there. In fact, a more realistic approximation would be that the liquid velocity equals the core velocity U_0. If indeed the transition from primary to secondary atomization occurs at some specific liquid to gas relative velocity ΔU_c, then there should be a line η_c which denotes the separation between these two regions of different atomizing characteristics. Hence, if it is assumed that the velocity of the liquid for $\eta_c \leqslant \eta \leqslant 1$ is equal to U_0 then along η_c the gas velocity will equal $U_0 - \Delta U_c$. Substituting these values into Eq. (4) yields

$$\bar{U} = 1 - \Delta\bar{U}_c(1 - C_d) \tag{5}$$

which, through the use of Eqs. (1) and (2), can be put into the form

$$\eta_c^3 - 2\eta_c^{3/2} + \Delta\bar{U}_c(\eta_c - 1) + 1 = 0 \tag{6}$$

The solution of Eq. (6) will give the value of η_c for any particular $\Delta\bar{U}_c$ (which of course depends on the fluid properties and on U_0). Equation (6) has been solved numerically and the result is shown in Fig. 2. Only the region in the vicinity of $\eta = 1$ is of importance, since the assumption that $U_d = U_0$ would not be valid far from the inner boundary.

To discuss the implications of Fig. 2, a useful reference is the value of η for $r = r_0$ (i.e., η_0). In Ref. 3, it has been shown that all η = const are essentially straight lines. Hence η_0 is virtually independent of x and depends only on the relative densities of the liquid and gas. Its behaviour, as calculated in Ref. 3, is also shown in Fig. 2.

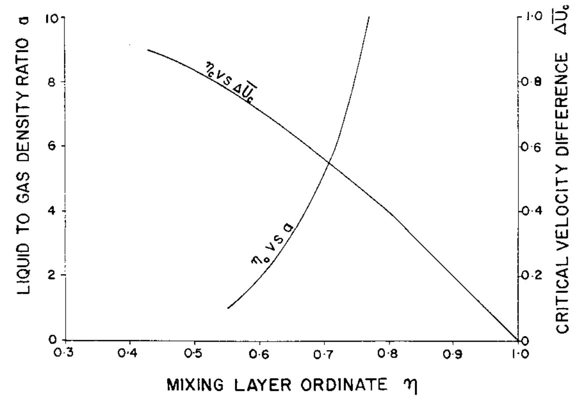
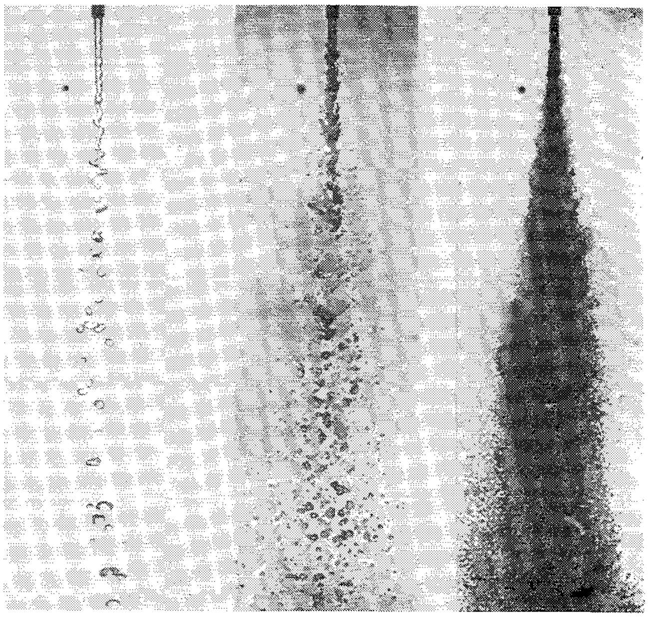


Fig. 2 Variation of orifice radius ordinate and ordinate along which relative velocity equals critical velocity difference.



a) Primary atomization, U_0 = 20 cm/sec **b) Primary and secondary atomization, U_0 = 100 cm/sec** **c) Secondary atomization, U_0 = 300 cm/sec**

Fig. 3 Influence of jet velocity on atomization characteristics (spray formed by the injection of liquid carbon dioxide into a saturated mixture of CO_2 and N_2 gas; chamber pressure = 900 psig, chamber temperature = 22°C).

An examination of Fig. 2 indicates some interesting features about the atomization process as it takes place in the initial region. For secondary atomization to occur at all η_c must be greater than η_0. For a given gas density, the likelihood of η_c being greater than η_0 increases as liquid density decreases. η_c will be less than η_0 when, for example, U_0 is small and/or ΔU_c is large. If $\Delta\bar{U}_c$ is such that $\eta_0 < \eta_c < 1$, both primary and secondary atomization will occur. In this case, both large lumps of liquid and small droplets should appear in the main region, the larger globules being generally confined to the central portion of the spray. The limiting case of $\eta_c = 1$ will occur when $\Delta\bar{U}_c$ is small, i.e., surface tension small, and gas density and U_0 large. In this case the spray should consist entirely of very small droplets. These three possible solutions are shown in the three photographs of Fig. 3. In the third case, which is that of most practical interest in a high-pressure environment, the length of the jet core x_i represents quite well the atomization or jet breakup length.

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